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NATURAL CONVECTION COOLING TRANSIENTS

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Abstract—An integral theory of natural convection transients, which has been successfully compared with experimental results, is used to study the transient temperature response of an element, having thermal capacity, whose energy input is abruptly terminated. An element thermal capacity parameter arises in the analysis whose value indicates the boundary between the regimes of true convection transients and essentially quasi-static processes. This value has been determined by calculations.

NOTATION

a, Prandtl number dependent constant;
b, generalizing factor for time;
c, specific heat of fluid;
c'', thermal capacity of element per unit surface area;
g, local gravitational acceleration;
h, local surface coefficient;
k, thermal conductivity;
q'', instantaneous energy generation rate per unit of element surface area;
u_g, a velocity, proportional to that in steady state;
u_m, instantaneous local velocity maximum;
L, height of element;
M_0, derivative of the generalized temperature distribution;
Gr, conventional Grashof number, based upon L and average temperature excess at beginning of transient, absolute value;
Gr^*, modified Grashof number, based upon L and surface flux at beginning of transient, absolute value;
Pr, Prandtl number;
Q, constant related to the element storage capacity, equation (8);
S, Prandtl number dependent constant;
t, temperature;
T, generalized time variable, equation (7);
U, Prandtl number dependent constant;
W, Prandtl number dependent constant;
Y $= \Delta_\theta/\Delta_{\theta,\infty}$.

Greek symbols
α, thermal diffusivity of fluid;
β, coefficient of thermal expansion of fluid;
δ_θ, thickness of thermal boundary layer;
Δ_θ $= \delta_\theta/L$.
θ, local temperature excess $(t - t_r)$;
θ_m, instantaneous local temperature maximum (or minimum);
μ, fluid absolute viscosity;
ρ, density of fluid;
τ, time;
ν, fluid kinematic viscosity;
χ, u_m/u_g;
ψ, $\theta_m/\theta_{m,\infty}$;
$\bar{\theta}_m$, $\bar{\psi}$, $\bar{Y}$, and $\bar{\chi}$ are instantaneous average values over the height of the element.

Subscripts
∞, at the beginning of the transient;
e, exponential response;
r, in the remote fluid;
s, quasi-static.

INTRODUCTION

IN A PREVIOUS paper [1] the writer presented a "double integral" method for transients in natural convection in a single phase fluid. This theory is applicable to surfaces parallel to the body force and includes the effect of thermal capacity in the element which has the convecting surface, although conduction internal to the element parallel to the surface is assumed negligible. It is assumed that this thermal capacity element is subject to an energy input condition. The energy input may be variable in time and either positive, zero, or negative. However, the assumptions of the analysis suggest that the theory should not be applied indiscriminately for transients resulting from a rapid oscillation in the rate of energy input.

The resulting differential equations have been solved for a step in energy input, in [1] for an element of zero element thermal capacity and in [2] for non-zero element thermal capacity. The calculations in [2] indicate that, from the point of view of element temperature response, there are three regimes for the convection process in the fluid. The regimes are: essentially one-dimensional conduction, a true convection transient, and essentially quasi-static. The calculations indicate the limits of these regimes in terms of two values of a thermal capacity parameter Q which arises in the analysis. The total range for a true convection transient is a variation of Q of only one order of magnitude.

The responses calculated in [2] were compared in [3] with measured convection transients in air and in water for a wide range of conditions. The measured responses were within 5 per cent of the predictions of the theory and, in addition, substantiated the conclusions concerning regimes.

Since the condition for a quasi-static response is not severe, even for a step in energy input, the limiting condition for quasi-static response for a linear increase in energy input rate was investigated. The calculated results are given in [4] as the limiting value of Q as a function of the time constant of the linear increase in energy input. We note that the step input in [2] is the limiting value of a zero time constant for the energy input variation.

Other analyses of natural convection transients are summarized in [1] and [2]. To date no exact treatments have appeared for realistic boundary conditions.

The present communication applies the theory of [1] to a circumstance which is very common in technology, namely the natural convection cooling or heating of an object having thermal capacity and which is initially at a temperature different from its surroundings. It is assumed that at the beginning of the transient process a steady state natural convection process is present with an energy input rate of q''_∞. There is no meaningful one-dimensional conduction regime for this case. The present consideration is an attempt to predict the conditions which separate essentially quasi-static responses from true convection transients.

The general differential equations relate the instantaneous value of the temperature, thermal layer thickness, and induced velocity maximum variables (ψ, Y, and χ) to generalized time T. The equations apply to vertical plates and to vertical cylinders under the conditions in which a laminar boundary analysis in Cartesian co-ordinates is permissible. The equations in terms of average values ($\bar\psi$, $\bar Y$, and $\bar\chi$), averaged over the height of the element, are written as

$$\frac{\bar\psi}{\bar Y} - a\frac{\mathrm{d}}{\mathrm{d}T}(\bar\psi\,\bar Y) - \bar Y\,\bar\chi\,\bar\psi = 0 \qquad (1)$$

$$S\,\bar\psi\bar Y - U\frac{\bar\chi}{\bar Y} - \frac{\mathrm{d}}{\mathrm{d}T}(\bar\chi\,\bar Y) - W\bar Y\,\bar\chi^2 = 0 \qquad (2)$$

$$\frac{\bar\psi}{\bar Y} = \frac{q''}{q''_\infty} - Q\frac{\mathrm{d}\bar\psi}{\mathrm{d}T} \qquad (3)$$

where the constants, S, U, W,† and a, depend only upon Prandtl number. The thermal flux quantities, q'' and q''_∞, are the instantaneous and asymptotic (or initial) values of the rate of energy input to the element per unit of surface area. The dependent variables $\bar\psi$, $\bar Y$, and $\bar\chi$, the generalized time T, and the generalized thermal capacity variable Q are defined as follows:

$$\bar\psi = \frac{\bar\theta_m}{\theta_{m,\infty}} \qquad (4)$$

$$\bar Y = \frac{\bar\Delta_\theta}{\Delta_{\theta,\infty}} \qquad (5)$$

$$\bar\chi = \bar u_m/u_g \qquad (6)$$

$$T = \frac{\alpha\tau}{L^2}(b\,Gr^*\,Pr)^{2/5} \qquad (7)$$

$$Q = \frac{c''}{\rho c L M_0}(b\,Gr^*\,Pr)^{1/5} \qquad (8)$$

* Note that $S - U - W = 0$, from steady-state considerations.

where the Prandtl number dependent quantities M_0 and b are known from [2] and the other quantities are defined in the listing of notation. The modified Grashof number is

$$Gr^* = \frac{g\beta q''_\infty L^4}{k\nu^2}$$

CALCULATIONS

For the problem under consideration here $q'' = 0$ for $\tau > 0$ and equations (1), (2), and (3) may be reduced to:

$$\bar{\psi}'' = \frac{\bar{\psi}'}{a\bar{\psi}^2}\left(2a\bar{\psi}\bar{\psi}' + \bar{\chi}\bar{\psi}^2 - Q^2\bar{\psi}'^2\right) \quad (9)$$

$$\bar{\chi}' = S\bar{\psi} - UQ^2\frac{\bar{\chi}\bar{\psi}'^2}{\bar{\psi}^2} - W\bar{\chi}^2 - \frac{\bar{\chi}\bar{\psi}'}{\bar{\psi}} + \frac{\bar{\chi}\bar{\psi}''}{\bar{\psi}'} \quad (10)$$

at $$T = 0: \quad \bar{\psi} = \bar{\chi} = 1 \quad (11)$$

where the prime indicates differentiation with respect to T. We note also that $\bar{\psi}' = \bar{\chi}' = Q^{-1}$ at $T = 0$.

The quasi-static solution $\bar{\psi}_s$ may be obtained most simply by neglecting the time rate of change of energy and momentum in the convection layer in equations (1) and (2), i.e. the time derivatives in those equations. The differential equation and solution for the condition of (11) are:

$$-Q\bar{\psi}'_s = \bar{\psi}_s^{-5/4} \quad (12)$$

$$\bar{\psi}_s = \frac{1}{\left(\frac{T}{4Q} + 1\right)^4} \quad (13)$$

This result is independent of Prandtl number. The quasi-static decay of element temperature is shown in Fig. 1. Also shown is the exponential decay with the same initial slope, i.e. $\bar{\psi}_e = e^{-(T/Q)}$. The reason for the difference is that the quasi-static, as calculated here, takes into account the variation of the convection coefficient with time, note the 5/4 exponent of $\bar{\psi}_s$ in equation (12). The simple exponential decay may be sufficiently close for some purposes.

The full equations [(9) and (10)] were numerically integrated by a Runge–Kutta technique at a tolerance of 10^{-5} for values of the constants which apply for the Prandtl number of air, $Pr = 0{\cdot}72$, i.e. $a = 0{\cdot}2$; $b = 40{\cdot}25 \times 10^{-4}$; $S = 16{\cdot}128$; $U = 9{\cdot}242$; $W = 6{\cdot}886$. These values are based upon the steady-state forms of the temperature and velocity distributions [2]. Calculations were carried out for values of Q of 1·0, 0·1, and 0·01. An abstract of the results is tabulated below. The responses are plotted against T/Q on Fig. 2. The quasi-static response is also shown.

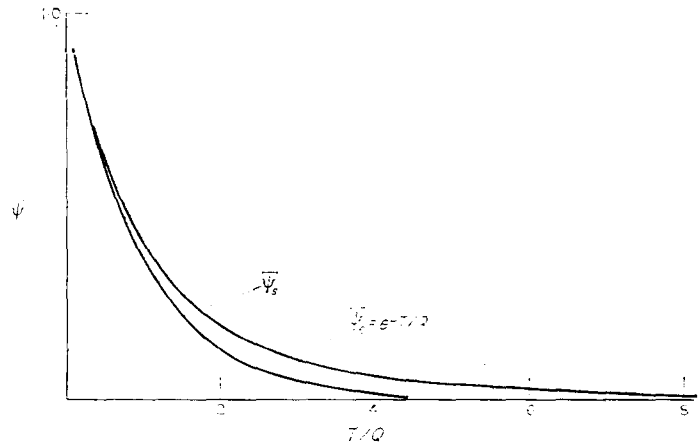

FIG. 1. The quasi-static and exponential responses.

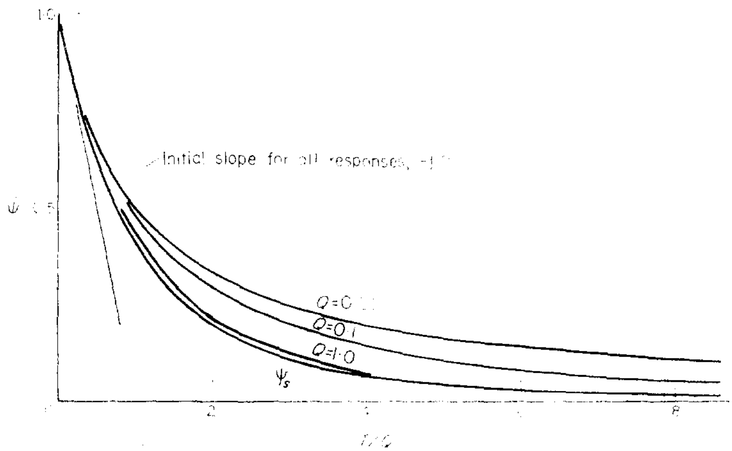

FIG. 2. Convection transient responses compared with the quasi-static responses.

Table 1. Convection transient and quasi-static solutions

T/Q	$\bar{\psi}$ for $Q =$ 0·01	$\bar{\psi}$ for $Q =$ 0·1	$\bar{\psi}$ for $Q =$ 1·0	$\bar{\psi}_s$ equation (13)
0·01		0·9901	0·9901	0·9901
0·05		0·9524	0·9523	0·9515
0·10	0·9091	0·9090	0·9083	0·9060
0·15		0·8694	0·8673	0·8631
0·2	0·8333	0·8329	0·8287	0·8227
0·3	0·7691	0·7681	0·7578	0·7488
0·4	0·7141	0·7120	0·6943	0·6830
0·5	0·6664	0·6630	0·6373	0·6243
0·6	0·6246	0·6196		0·5718
0·8	0·5548	0·5463		0·4823
1·0	0·4990	0·4863	0·4267	0·4096
1·2	0·4532	0·4363	0·3675	0·3501
1·4	0·4150	0·3939		0·3011
1·6	0·3830	0·3575		0·2603
1·8	0·3550	0·3259	0·2426	0·2262
2·0	0·3309	0·2983		0·1975
2·5	0·2828	0·2423		0·1434
3·0	0·2467	0·2000		0·1066
3·5	0·2186	0·1673		0·0809
4·0	0·1961	0·1415		0·0625
4·5	0·1777	0·1208		0·0490
5·0	0·1624	0·1041		0·0390
6·0	0·1383			0·0256
7·0	0·1203			0·0174
8·0	0·1063			0·0123

CONCLUSIONS

Comparison of the transient responses with the quasi-static shows that the value of Q delimits the regimes of transient and quasi-static behavior. For $Q = 1{\cdot}0$, $\bar{\psi} - \bar{\psi}_s$ remains less than 0·02 over the whole range of T/Q. Therefore, one may say that all processes for which $Q \geqslant 1{\cdot}0$ have an essentially quasi-static response.

For smaller values of Q the difference is much greater, particularly if one is interested in the time necessary, for example, to cool an element to essentially ambient temperature. To obtain a value of $\bar{\psi}$ of 0·1 for a circumstance having $Q = 0{\cdot}1$, the time interval for the transient result is 65 per cent greater than that estimated by the quasi-static. For $Q = 0{\cdot}01$, the time interval is 170 per cent greater. This difference could be of great importance in, for example, electronic element or reactor element transients and in circumstances for which thermally induced stresses are an important aspect of design.

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REFERENCES

1. B. GEBHART, Transient convection from vertical elements, *Trans. ASME*, **83C**, 61 (1961).
2. B. GEBHART, Transient natural convection from vertical elements—appreciable thermal capacity, *Trans. ASME*, **85C**, 10 (1963).
3. B. GEBHART and D. E. ADAMS, Measurements of transient natural convection on flat vertical surfaces, *Trans. ASME*, **85C**, 25 (1963).
4. B. GEBHART, Transient natural convection for vertical elements for a time dependent internal energy generation—Appreciable thermal capacity, *Int. J. Heat Mass Transfer* **6**, 951 (1963).

Résumé—Une théorie intégrale des transitoires par convection naturelle, que l'on a comparée avec succès aux résultats expérimentaux, est utilisée pour l'étude de la réponse transitoire de température d'un élément, ayant une capacité thermique, dont l'alimentation en énergie est arrêtée brutalement. Un paramètre de capacité thermique de l'élément apparaît dans l'analyse; sa valeur indique la limite entre les régimes de vrais transitoires de convection et les processus essentiellement quasi-statiques. On a déterminé cette valeur par des calculs.

Zusammenfassung—Eine Integralmethode der instationären freien Konvektion, die erfolgreich mit Versuchsergebnissen verglichen wurde, dient dazu, die instationäre Temperaturänderung eines Elements bestimmter Wärmekapazität zu untersuchen, wenn die Energiezufuhr plötzlich unterbrochen wird. Ein Parameter für die Wärmekapazität des Elements erscheint in der Analyse; sein Wert gibt die Grenze an zwischen den Bereichen instationärer Konvektion und vorwiegend quasistationärer Prozesse. Dieser Wert wurde berechnet.

Аннотация—Для изучения нестационарного изменения температуры элемента, имеющего теплоемкость, подвод тепла к которому внезапно прекращен, используется интегральная теория нестационарной свободной конвекции, дающая результаты, хорошо совпадающие с экспериментальными данными. Анализ показывает наличие параметра теплоемкости элемента, значение которого указывает границу между режимами действительно нестационарной конвекции и квази-стационарным. Эта величина была вычислена.